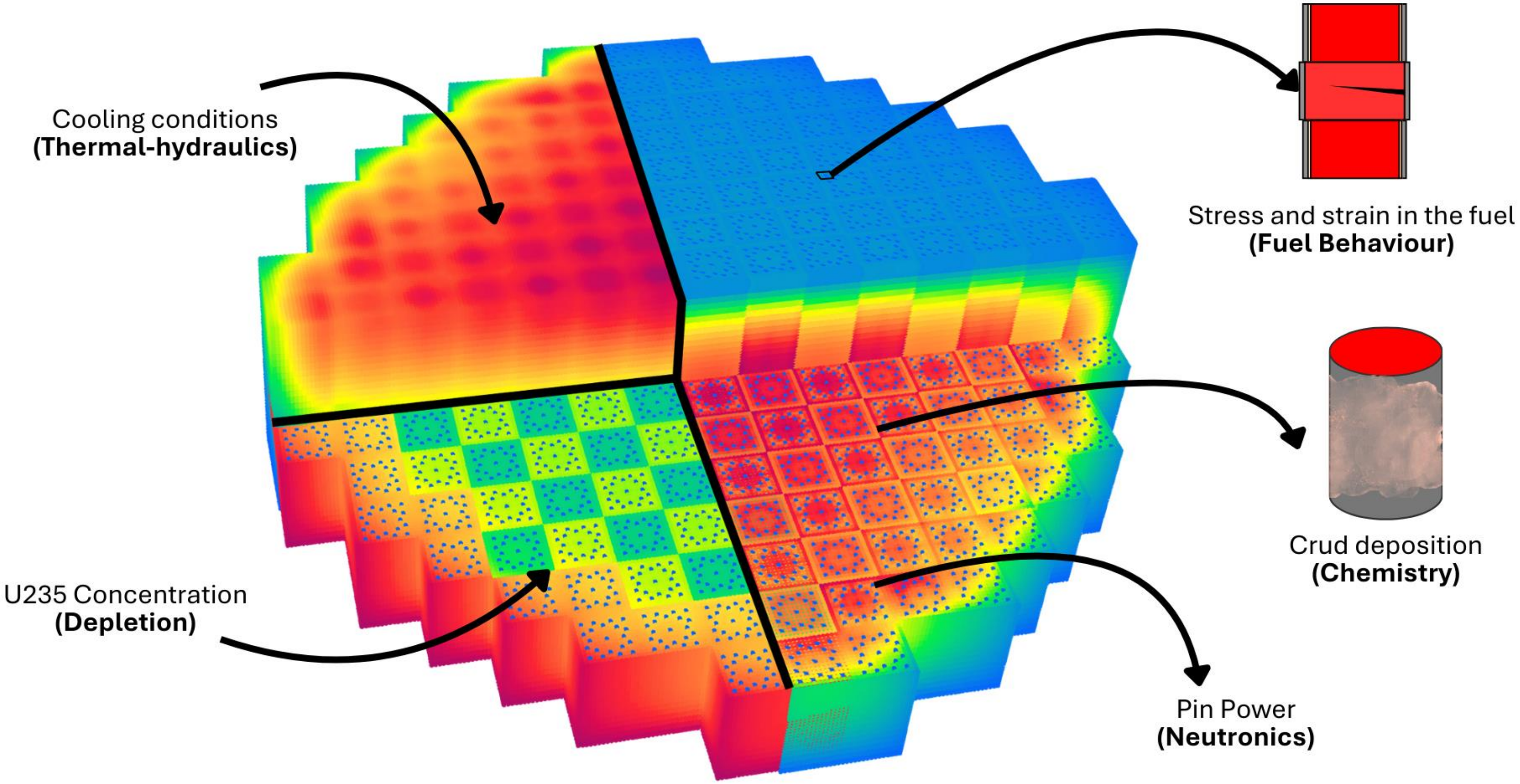


Physics of Nuclear Reactors

Extra: one group

Daniele Timpano

Milica Krstovic

Exercises of today

SOLUTION OF THE ONE-GROUP DIFFUSION THEORY IN MULTIPLYING MEDIA (EXTRA)

17.1 Spherical graphite reactor
17.2 Infinite moderator
17.3 1-group expression for reflector savings
17.4 Minimal critical volume of a cylindrical reactor

Divide in groups of 5:

- *I will leave you 10 minutes for Exercise 1*
- *A short break to go through the exercise and reflect on diffusion theory*
- *I will leave you 10 minutes for Exercise 4*
- *We will go through the solution together*

1. Spherical graphite reactor

Exercise description:

Consider a reactor made of graphite, in which one has a highly diluted quantity of pure ^{235}U, with a 0.0035% concentration ($N_{fuel}/N_{mod}=3.5\cdot 10^{-5}$).

Calculate the critical radius of this installation, applying modified 1-group diffusion theory (replace L^2 by $M^2 = L^2 + \tau$). The medium is sufficiently moderated for the reactor to be considered thermal. Use the following data:

U-235	Graphite
$\sigma_s = 8.3$ b	$\sigma_s = 4.8$ b
$\sigma_c = 108$ b	$\sigma_a = 0.0048$ b
$\sigma_f = 580$ b	$\tau_{th} = 368$ cm^2
$\eta_c = 2.068$	$\rho = 1.6$ g/cm^3

N.B.: The graphite in this exercise is industrial-grade, its absorption cross-section being significantly greater than that for pure carbon; the data provided takes the impurities into account.

Knowledge to be applied: $D = \frac{1}{3\Sigma_t}$, $L^2 = \frac{D}{\Sigma_a}$, $M^2 = L^2 + \tau$, $k_\infty = \eta(U)f$, $B_m^2 = B^2 = \frac{k_\infty - 1}{M^2}$,

$d = 0.71\lambda_t$

Expected results: $R_c = 98.50$cm

1. Spherical graphite reactor

- One uses the **one-group** diffusion equation for the stationary case (critical reactor)

$$D\nabla^2\phi(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) + S(\vec{r}) = 0$$

- The source is coming from fissions: $S(\vec{r}) = Q_f(\vec{r}) = \bar{\nu}\Sigma_f(\vec{r})\phi(\vec{r})$

- Thus $D\nabla^2\phi + (\bar{\nu}\Sigma_f - \Sigma_a)\phi = 0 \quad \Longrightarrow \quad \nabla^2\phi + \dfrac{\dfrac{\bar{\nu}\Sigma_f}{\Sigma_a} - 1}{\dfrac{D}{\Sigma_a}}\phi = 0$

$$\nabla^2\phi + \frac{k_\infty - 1}{L^2}\phi = 0$$

$$\nabla^2\phi + B_m^2\phi = 0$$

One-group Reactor Equation

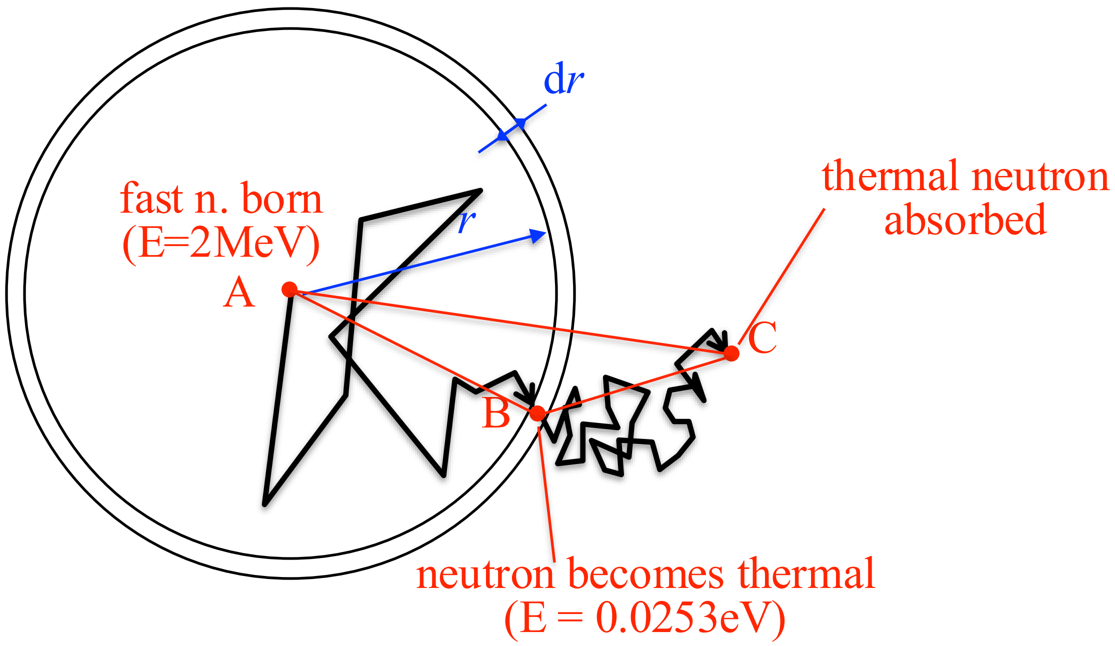
$$B_m^2 = \frac{k_\infty - 1}{\cancel{L^2}\ M^2}$$

Material Buckling

(depends only material properties)

What changes in modified diffusion theory and why?

1. Spherical graphite reactor



Age to thermal : $\tau_T = \frac{1}{6}\overline{|AB|}^2$

Diffusion area : $L^2 = \frac{1}{6}\overline{|BC|}^2$

Migration area : $M^2 = \tau_T + L^2$

What changes in modified diffusion theory and why?

1. Spherical graphite reactor

The **number density of graphite in the sphere is**:

$$N_{\text{graph}} \simeq \frac{\rho N_A}{\mathrm{M}_{\text{graph}}} = \frac{1.6\ \mathrm{g/cm3} \times 6.022 \times 10^{23}\ \mathrm{atom/mol}}{12.0\ \mathrm{g/mol}} = 8.0 \times 10^{22}\mathrm{cm}^{-3}$$

Using the information on **fuel to moderator ratio**:

$$N_{\text{fuel}} = N_{\text{graph}} \times (\mathrm{F/M}) = 8.0 \times 10^{22} \times 3.5 \cdot 10^{-5} = 2.8 \times 10^{18}\mathrm{cm}^{-3}$$

You can now compute the **XSs of the mixture**:

$$\Sigma_a = N_c\sigma_{a,c} + N_m\sigma_{a,m} = 2.8 \times 10^{18} \times 688 \times 10^{-24} + 8 \times 10^{22} \times 0.0048 \times 10^{-24} = 2.319 \times 10^{-3}\mathrm{cm}^{-1}$$

$$\Sigma_s = N_c\sigma_{s,c} + N_m\sigma_{s,m} = 2.8 \times 10^{18} \times 8.3 \times 10^{-24} + 8 \times 10^{22} \times 4.8 \times 10^{-24} = 0.385\mathrm{cm}^{-1}$$

$$\Sigma_t = \Sigma_s + \Sigma_a = 0.388\mathrm{cm}^{-1}$$

1. Spherical graphite reactor

Compute the **migration area**:

$$D=\frac{1}{3\Sigma_t}=0.8597\text{ cm},\ L^2=\frac{D}{\Sigma_a}=370.7\text{ cm}^2\text{, and }M^2=L^2+\tau=738.72\text{ cm}^2$$

Differently from the past weeks, we have a system with both **fuel and moderator**. But there is no fertile materials, so *no resonances* and not *fast fission* $\mathrm{p}=\epsilon=1$. Applying the four factors formula reduces to:

$$\mathrm{k}_\infty=\eta f=\eta\frac{N_c\sigma_{a,c}}{\Sigma_a}==2.068\times0.83380=1.72429$$

Note: The calculations show that the absorption cross-section is very much smaller than the total cross-section (0.00231 cm^{-1}, against 0.386), and that the mean free path is almost negligible in comparison with the size of the reactor. Two conditions for the validity of diffusion theory are thus directly seen to be fulfilled. One also notes that the fuel hardly makes any contribution to the total cross-section of the mixture.

1. Spherical graphite reactor

Take a step back: what were the assumptions for diffusion theory?

- **Assumption** 1: *Scattering is isotropic in the laboratory coordinate system*

  Comment: More often scattering is isotropic in the center-of-mass system. In this case:

  $$D = \frac{1}{3\Sigma_s(1-\bar{\mu})} = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}, \text{ where } \bar{\mu} = \overline{\cos\theta} \text{ and } \theta \text{ is the scattering angle}$$

- **Assumption** 2: $\Sigma_a << \Sigma_s, \text{ or } \Sigma_t = \Sigma_a + \Sigma_s \simeq \Sigma_s$

  Comment: The Fick's law is not valid *in* or *close* to strong absorbers (e.g. fuel); when the absorber is diluted enough by moderator, Fick's law becomes valid

- **Assumption** 3: *The medium is infinite*

  Comment: Due to an attenuation factor $\exp(-\Sigma_s r)$ Fick's law is valid also in a finite medium (reactor), but in the interior, far enough from the outer surface (few λ's)

- **Assumption** 4:

  $\phi(\vec{r})$ is a slowly varying function of $\vec{r}$

1. Spherical graphite reactor

Apply the criticality condition for 1-group modified diffusion theory:

$$B_m^2 = B^2 = \left(\frac{\pi}{R + d}\right)^2 = \frac{k_\infty - 1}{M^2} = 9.8047 \times 10^{-4}\,\text{cm}^{-2}$$

The extrapolation distance is **d = 0.71 λ_t**, which here is only 1.83 cm., i.e. is indeed much smaller than the critical radius we find, viz. $\boldsymbol{R} =$ **98.50cm**.

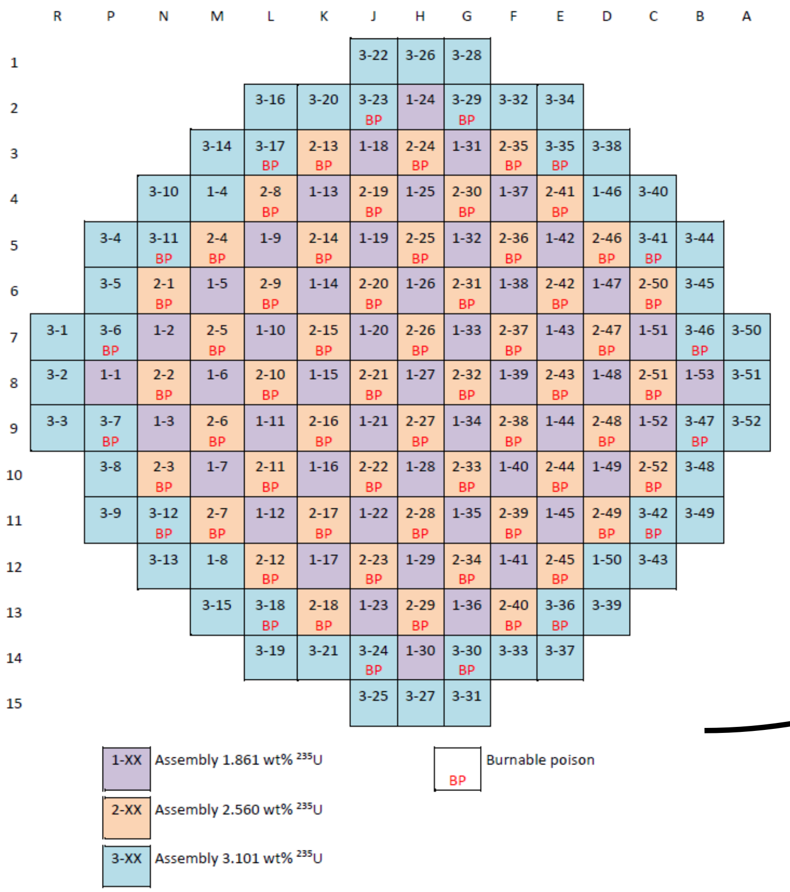
Take a step back:

Why is one-group diffusion theory not enough?

Limitations are in the assumptions!
Practical example in the next slides.

One group is not enough because:
Neutrons have a wide energy spectrum (born fast, fast fission / slow down, thermal fission)
Cross sections vary strongly with energy.
Neutron processes (fission, scattering, absorption) differ by energy region.

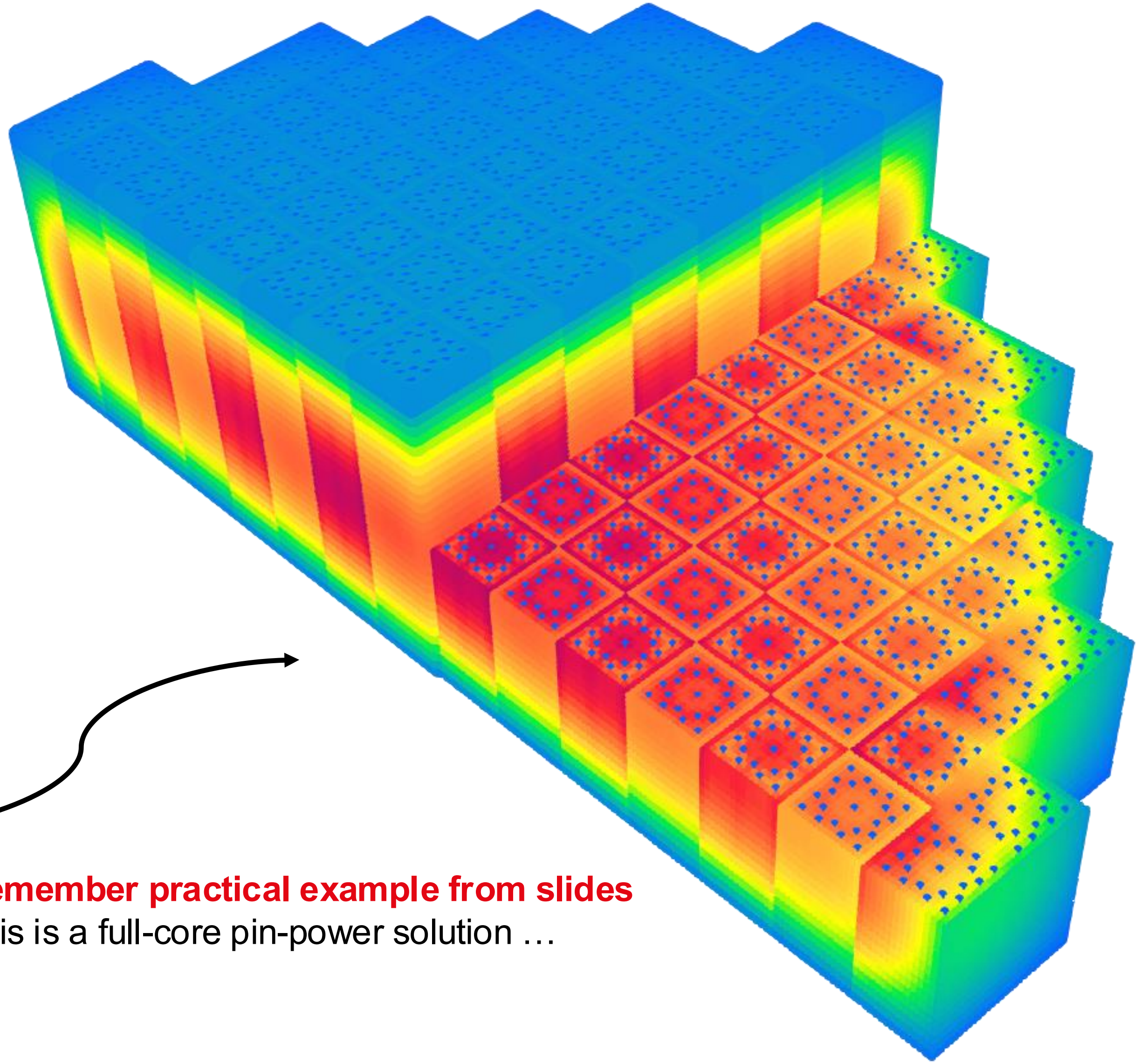
A real example of diffusion theory (?)



	R	P	N	M	L	K	J	H	G	F	E	D	C	B	A
1							3-22	3-26	3-28						
2					3-16	3-20	3-23 BP	1-24	3-29 BP	3-32	3-34				
3				3-14	3-17 BP	2-13 BP	1-18	2-24 BP	1-31	2-35 BP	3-35 BP	3-38			
4			3-10	1-4	2-8 BP	1-13	2-19 BP	1-25	2-30 BP	1-37	2-41 BP	1-46	3-40		
5		3-4	3-11 BP	2-4 BP	1-9	2-14 BP	1-19	2-25 BP	1-32	2-36 BP	1-42	2-46 BP	3-41 BP	3-44	
6		3-5	2-1 BP	1-5	2-9 BP	1-14	2-20 BP	1-26	2-31 BP	1-38	2-42 BP	1-47	2-50 BP	3-45	
7	3-1	3-6 BP	1-2	2-5 BP	1-10	2-15 BP	1-20	2-26 BP	1-33	2-37 BP	1-43	2-47 BP	1-51	3-46 BP	3-50
8	3-2	1-1	2-2 BP	1-6	2-10 BP	1-15	2-21 BP	1-27	2-32 BP	1-39	2-43 BP	1-48	2-51 BP	1-53	3-51
9	3-3	3-7 BP	1-3	2-6 BP	1-11	2-16 BP	1-21	2-27 BP	1-34	2-38 BP	1-44	2-48 BP	1-52	3-47 BP	3-52
10		3-8	2-3 BP	1-7	2-11 BP	1-16	2-22 BP	1-28	2-33 BP	1-40	2-44 BP	1-49	2-52 BP	3-48	
11		3-9	3-12 BP	2-7 BP	1-12	2-17 BP	1-22	2-28 BP	1-35	2-39 BP	1-45	2-49 BP	3-42 BP	3-49	
12			3-13	1-8	2-12 BP	1-17	2-23 BP	1-29	2-34 BP	1-41	2-45 BP	1-50	3-43		
13				3-15	3-18 BP	2-18 BP	1-23	2-29 BP	1-36	2-40 BP	3-36 BP	3-39			
14					3-19	3-21	3-24 BP	1-30	3-30 BP	3-33	3-37				
15							3-25	3-27	3-31						

1-XX	Assembly 1.861 wt% ^{235}U
2-XX	Assembly 2.560 wt% ^{235}U
3-XX	Assembly 3.101 wt% ^{235}U
BP	Burnable poison

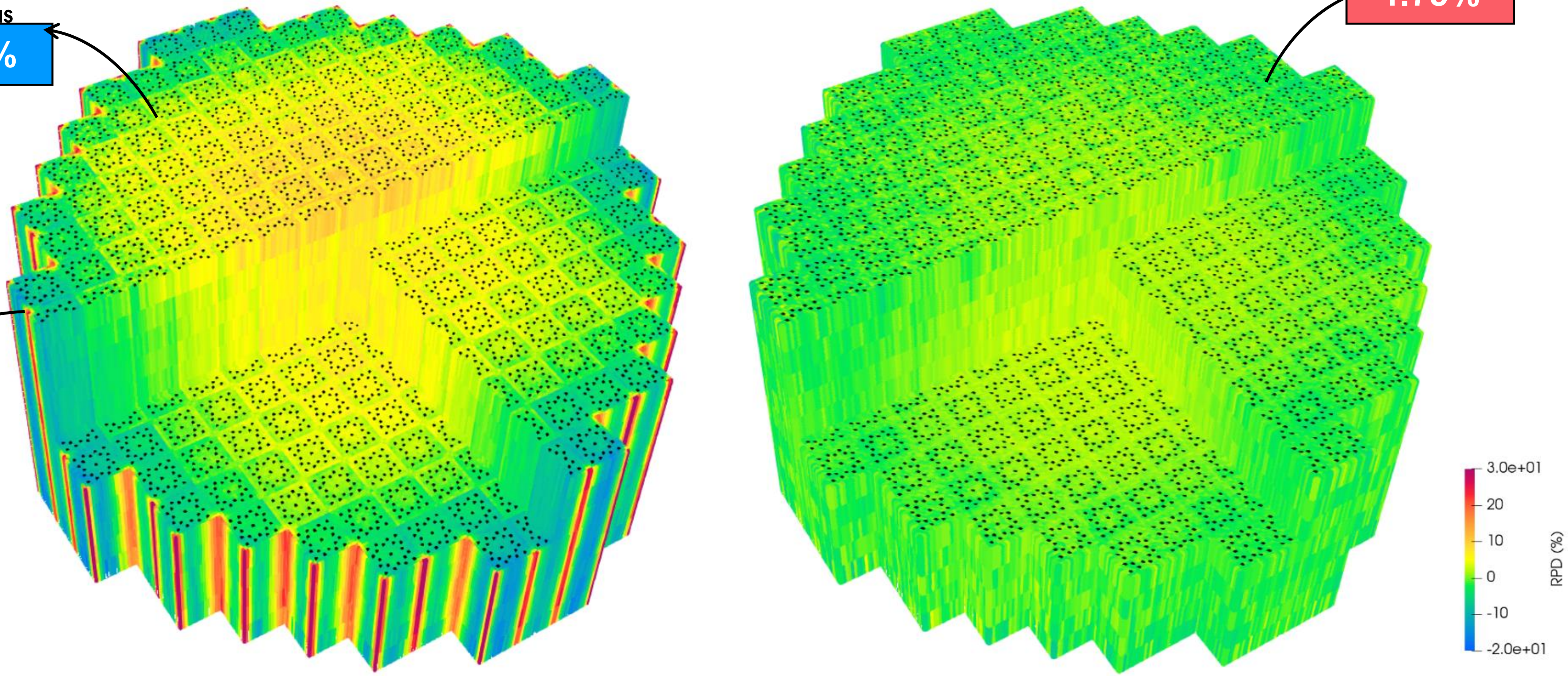
Figure 2.3. Layout of fuel assemblies and BPs in TP3 cycle 1.



Remember practical example from slides

This is a full-core pin-power solution …

A real example of diffusion theory (?)



3D Pin RMS

5.10%

Is this really JUST diffusion? (next slide)

“Poor” performance close to the boundaries! Assumption 3 and 4 …

3D Pin RMS

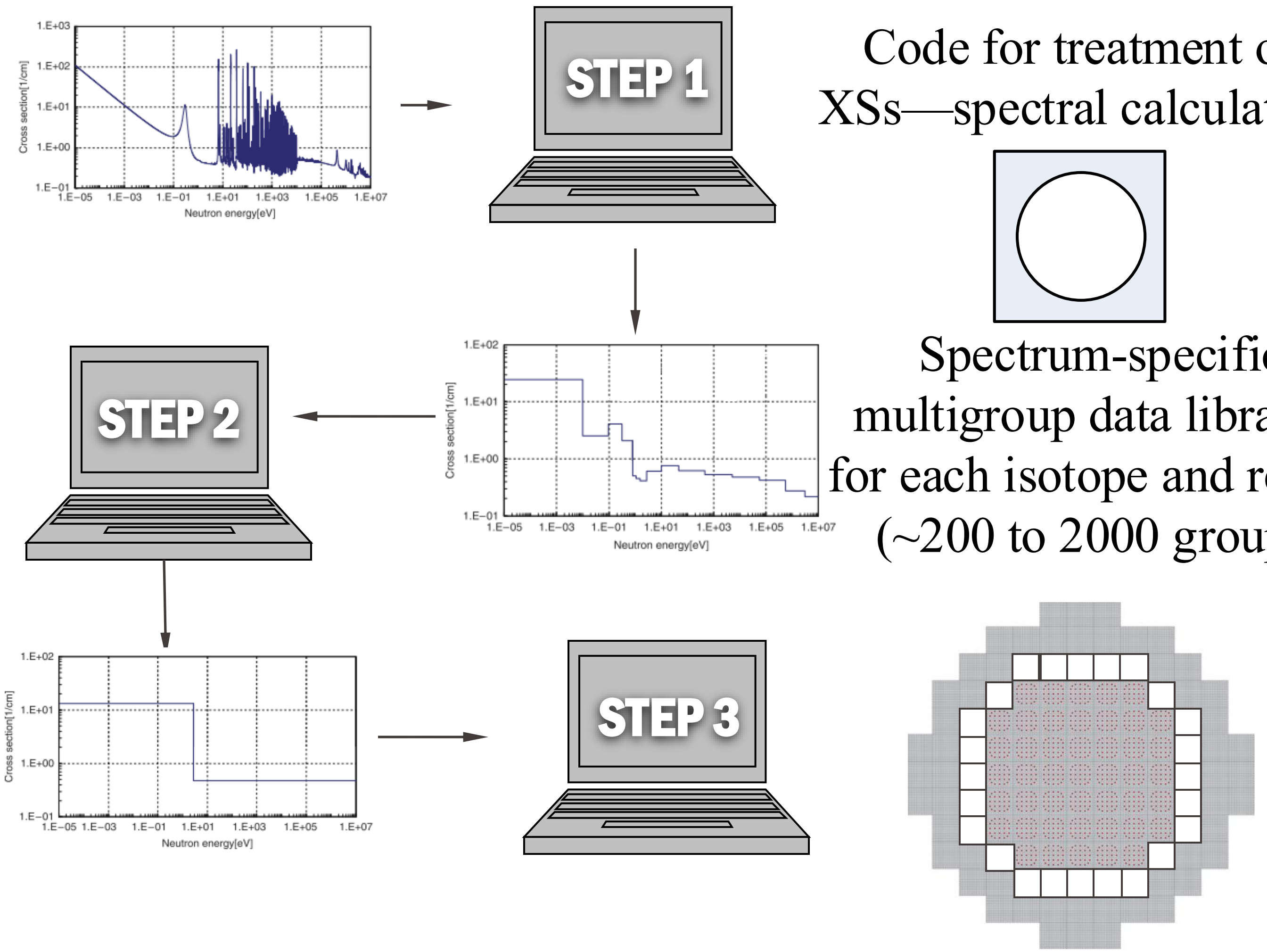
1.75%

Polaris/PARCS ¦ 2-step approach + diffusion
Δk_inf = 245.3 pcm

MPACT ¦ On-the-fly resonance + transport
Δk_inf = 61.3 pcm

Comparison with reference solution from Serpent Monte-Carlo 3D simulation.

Practical reactor calculations are not only diffusion



Basic nuclear data for each isotope: XSs versus *E*—thousands points

Code for treatment of XSs—spectral calculation

Spectrum-specific multigroup data libraries for each isotope and region (~200 to 2000 groups)

Transport-theory lattice code

Homogenized XSs for reactor zones (~ 2 to 30 groups)

3D reactor calculations with multigroup diffusion code

Goal: Determine reactor parameters: k-eff, power distribution, safety coefficients, etc.

4. Minimal critical volume of a cylindrical reactor

Exercise description:

Derive an expression for the minimal critical volume of a cylindrical reactor, in terms of its material buckling. Assume the extrapolation distance to be negligible in comparison with the dimensions of the system.

Knowledge to be applied: $V = \pi R^2 H$, $B_m^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$, $\frac{\partial V}{\partial H}\square_{V_{\min}} = \frac{\partial V}{\partial R}\square_{V_{\min}} = 0$

Expected results: $V_{\min} \simeq \frac{148.3}{B_m^3}$

Hint 1: apply the criticality condition to express the radius as a function of height and material buckling

Hint 2: express the volume as a function of the height only

Hint 3: minimize the volume

Hint 4: substitute the condition for H and R in the volume expression

4. Minimal critical volume of a cylindrical reactor

1. Neglecting d, $V = \pi R^2 H$ and $\boldsymbol{B_m^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2}$. Accordingly: $\boldsymbol{R^2 = \frac{(2.405)^2}{B_m^2 - \left(\frac{\pi}{H}\right)^2}}$.

2. Substituting for R^2 into the volume: $\boldsymbol{V = \frac{\pi(2.405)^2 H}{B_m^2 - \left(\frac{\pi}{H}\right)^2}}$.

3. For minimal V: $\frac{\partial V}{\partial H} \quad V_{\text{min}} = \frac{\partial V}{\partial R} \quad V_{\text{min}} = 0$

$$\boldsymbol{\frac{\partial V}{\partial H} = \frac{\left(B_m^2 - \left(\frac{\pi}{H}\right)^2\right)\pi(2.405)^2 - \pi(2.405)^2 H \frac{2\pi^2}{H^3}}{\left(B_m^2 - \left(\frac{\pi}{H}\right)^2\right)^2}} = 0 \text{ and so } B_m^2 - \left(\frac{\pi}{H}\right)^2 = \frac{2\pi^2}{H^2} \Rightarrow H = \frac{\sqrt{3}\pi}{B_m}$$

4. Substituting H into the expression for R^2: $\boldsymbol{R^2 = \frac{(2.405)^2}{B_m^2 - \frac{B_m^2}{3}} = \frac{3\times(2.405)^2}{2B_m^2}}$.

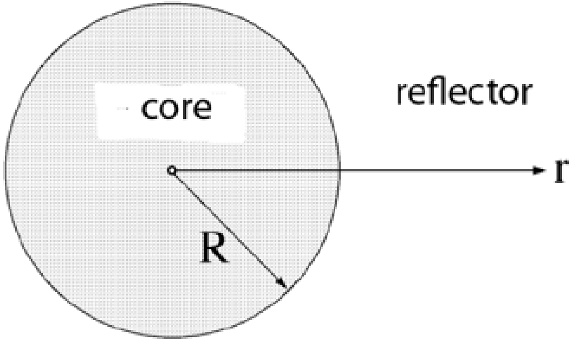
5. Finally: $\boldsymbol{V_{\text{min}} = \pi R^2 H = \pi \frac{3\times(2.405)^2}{2B_m^2}\frac{\sqrt{3}\pi}{B_m} \simeq \frac{148.3}{B_m^3}}$

2. Infinite moderator

Exercise description:

(a) Show that the critical condition (1-group) for a spherical reactor, surrounded by an infinite moderator, corresponds to the relation: $B_c R\cot(B_c R) - 1 = \frac{-D_r}{D_c}\left(\frac{R}{L_r} + 1\right)$, where B_c^2 is the material buckling of the core material, R is the core radius, D_c and D_r are the diffusion coefficients for core and reflector, respectively, and L_r is the diffusion length in the reflector.

(b) In terms of the power P of the reactor described above, derive the expressions for the absolute flux in the core and in the reflector.



Knowledge to be applied: $\frac{1}{r^2}\frac{d}{\mathrm{dr}}\left(r^2\frac{d\Phi_c}{\mathrm{dr}}\right) + B_c^2\Phi_c = 0 \Rightarrow \Phi_c = A\frac{\sin(B_c r)}{r} + C\frac{\cos(B_c r)}{r}$

$\frac{1}{r^2}\frac{d}{\mathrm{dr}}\left(r^2\frac{d\Phi_r}{\mathrm{dr}}\right) + \frac{1}{L_r^2}\Phi_r = 0 \Rightarrow \Phi_r = A'\frac{e^{-r/L_r}}{r} + C'\frac{e^{r/L_r}}{r}$

Expected results: (a) $B_c R\cot(B_c R) - 1 = \frac{-D_r}{D_c}\left(\frac{R}{L_r} + 1\right)$

(b) $\Phi_c = \frac{\mathrm{P}B_c^2}{4\pi E_f\Sigma_f\left(\sin(B_cR) - B_cR\cos(B_cR)\right)}\frac{\sin(B_c r)}{r}$, $\Phi_r = \frac{\mathrm{P}B_c^2\sin(B_cR)e^{R/L_r}}{4\pi E_f\Sigma_f\left(\sin(B_cR) - B_cR\cos(B_cR)\right)}\frac{e^{-r/L_r}}{r}$

2. Infinite moderator

(a)

For the core (1-group): $\frac{1}{r^2}\frac{d}{\mathrm{dr}}\left(r^2\frac{d\Phi_c}{\mathrm{dr}}\right)+B_c^2\Phi_c=0,\Rightarrow\Phi_c=A\frac{\sin(B_cr)}{r}+C\frac{\cos(B_cr)}{r}$

Because $\Phi_c \neq \infty$ at r=0 => C=0, so that $\Phi_c=A\frac{\sin(B_cr)}{r}$

For the reflector (1-group): $\frac{1}{r^2}\frac{d}{\mathrm{dr}}\left(r^2\frac{d\Phi_r}{\mathrm{dr}}\right)+\frac{1}{L_r^2}\Phi_r=0,\Rightarrow\Phi_r=A'\frac{e^{-r/L_r}}{r}+C'\frac{e^{r/L_r}}{r}$

Because $\Phi_r \neq \infty$ at r=∞ => C'=0, so that $\Phi_r=A'\frac{e^{-r/L_r}}{r}$.

The continuity conditions at the interface (r = R) yield, for flux and current, respectively:

$A\frac{\sin(B_cR)}{R}=A'\frac{e^{-R/L_r}}{R}$ and $AD_c\left(\frac{B_c\cos(B_cR)}{R}-\frac{\sin(B_cR)}{R^2}\right)=-A'D_r\left(\frac{1}{\mathrm{RL}_r}+\frac{1}{R^2}\right)e^{-R/L_r}$

Dividing these eq.: $D_c\left(B_c\cot(B_cR)-\frac{1}{R}\right)=-D_r\left(\frac{1}{L_r}+\frac{1}{R}\right)\Rightarrow B_cR\cot(B_cR)-1=\frac{-D_r}{D_c}\left(\frac{R}{L_r}+1\right)$

2. Infinite moderator

(b)

Reactor power: $P = \int_0^R E_f \Sigma_f \Phi(r) 4\pi r^2 \mathrm{dr} = \int_0^R E_f \Sigma_f A \frac{\sin(B_c r)}{r} 4\pi r^2 \mathrm{dr}$

which gives: $A = \frac{\mathrm{PB}_c^2}{4\pi E_f \Sigma_f \left(\sin(B_c R) - B_c R \cos(B_c R)\right)}$

Integrating by parts yields here $[-r\cos(r) + \sin(r)]$

From flux continuity equation: $A' = A\frac{\sin(B_c R)}{R}\frac{R}{e^{-R/L_r}} = A\sin(B_c R)e^{R/L_r}$

Thus, the desired expressions for the flux in core and reflector, respectively, are:

$$\Phi_c = \frac{\mathrm{PB}_c^2}{4\pi E_f \Sigma_f \left(\sin(B_c R) - B_c R\cos(B_c R)\right)} \frac{\sin(B_c r)}{r}$$

$$\Phi_r = \frac{\mathrm{PB}_c^2 \sin(B_c R) e^{R/L_r}}{4\pi E_f \Sigma_f \left(\sin(B_c R) - B_c R\cos(B_c R)\right)} \frac{e^{-r/L_r}}{r}$$